Centre de Recherches Mathématiques
CRM Proceedings and Lecture Notes
Volume **47**, 2008

Sculptural Presentation of the Icosahedral Rotation Group

George W. Hart

Abstract. A 1.2 meter diameter wooden geometric sculpture was assembled by participants at the McKay conference. Its sixty components are in one-to-one correspondence with the elements of the icosahedral rotation group. We constructed it on the first day, so the sculpture served as a centerpiece in the common room until we disassembled it on the last day. It presents the beauty of symmetry in a visual form, analogous to the mathematical beauty presented in more abstract form in talks in the conference hall.

The conference *Groups and Symmetries*: *From the Neolithic Scots to John McKay* was a meeting of participants with a keen interest in groups and symmetry. John McKay invited me to participate in an artistic role, by contributing a mathematical construction to the event. As a sculptor of geometric forms, I enjoy creating sculptures that are assembled as a "barn raising" by a group of participants. Examples of such works can be seen on my web pages [2]. These activities are celebrations of mathematical beauty in a sculptural form.

For the McKay conference at the CRM in Montreal, I provided a set of laser-cut wooden parts that assemble into a 1.2 meter diameter hanging sculpture. The result has sixty icosahedrally arranged components and can be seen as a physical presentation of the icosahedral symmetry group. Shown in Figure 1, the form is composed of identical cartoon-like plywood "people." Each is planar. The geometry and the pattern of connections are unintuitive, as adjacent "people" do not touch. Their limbs weave in or out past their neighbor's limbs and connect to parts slightly further away. So it takes some pondering and practice before the participants become fully comfortable with the structure.

In designing such a sculpture, I begin with a symmetrically arranged set of planes in space, calculate their lines of intersection, and devise a shape that connects one plane to another at the lines of intersection. I use a sculpture editing program [3] for implementing the symmetry transformations, visualizing the form, keeping track of the various lengths and angles, and outputting the part shape in a file format for laser cutting. For this sculpture, the sixty "people" are in the planes of Catalan's pentagonal hexacontahedron, the dual to the snub dodecahedron. This

2000 *Mathematics Subject Classification.* Primary ??; Secondary ??
This is the final form of the paper.

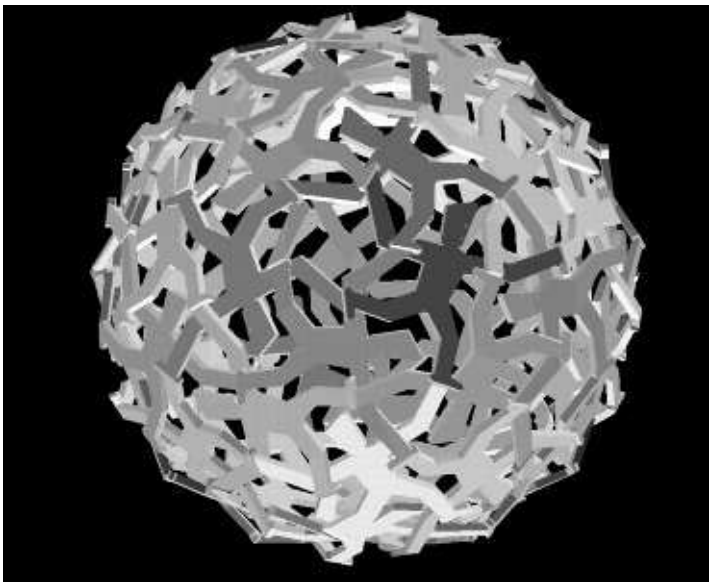

FIGURE 1. Computer model of sculpture's form, before fabricating components

is an isogonal polyhedron, meaning the symmetry group is transitive on the faces, so each component is surrounded by neighbors in an identical manner.

I began the assembly event by letting the participants explore the different ways that the parts might be connected. I did not give specific instructions except to say that the symmetry is icosahedral and transitive on the parts. Within a half hour the participants were able to try out and eliminate all the various ways of connecting the parts that were incompatible with these clues. For example, if seven parts seem to make a loop then the connection can not be correct, as the icosahedral group can not contain a subgroup of size seven. What remained was the correct method of assembly: every front arm connects to a front leg and every back arm connects to a back leg. Once that is known, the construction proceeds rapidly. Small groups of participants can make modules in parallel. Figure 2 shows a stage partway through the construction. The work is suspended from the ceiling so a half-dozen people can add to it from all sides at once.

At the wrists and feet, there is a hole for connecting parts together. They are neatly fastened with 120 aluminum connectors and 240 nylon thumbscrews. The screws are easily turned by hand, so no tools are required. The aluminum parts were drilled and tapped to accept the screws, bent to the proper dihedral angle, then spray painted to blend with the wood. Figure 3 shows a close-up of this hardware attached to one wooden arm.

After conceiving the design, I was able to cut the sixty Baltic birch plywood components on a laser cutter at the computer science and artificial intelligence laboratory at MIT. I am grateful to Erik Demaine and Martin Demaine for arranging this and for all their help. There we also test-fit the parts [1]. Laser cutting the components results in precise lengths and angles, ensuring that the parts fit exactly together while weaving in and out through each other in three dimensions. The disassembled parts are easily shipped from one venue to another. I have organized

FIGURE 2. Halfway through the construction

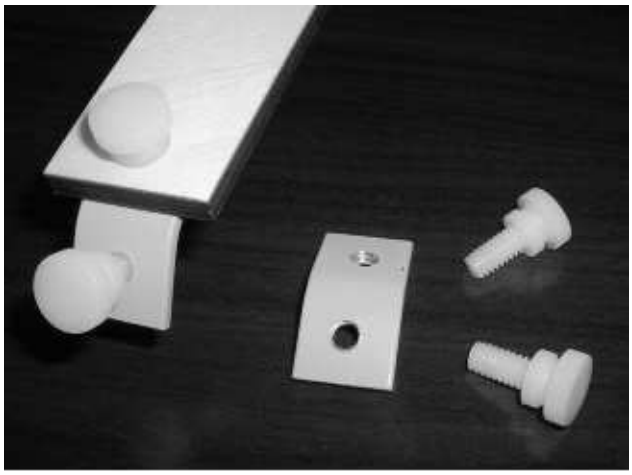

FIGURE 3. Aluminum connectors and nylon thumb screws

dozens of events like this at various colleges and conferences, using this same set of parts. About one hour is generally required for the assembly.

Figure 4 shows the Groups and Symmetries conference participants around the completed sculpture. We found it to be a unique bonding experience: an interdisciplinary activity combining the precision of geometric thinking, the beauty

FIGURE 4. Conference participants in group photo around sculpture

of wooden sculpture, the engineering of architectural structure, and the fun of an intricate assembly puzzle. I was honored to be able to present such an event at John McKay's celebration.

Acknowledgments. I thank John Harnad for arrangements which made my participation at the McKay conference possible. Portions of this material were presented at the Conference of the International Society of Art, Mathematics, and Architecture held at DePaul University, June 17 – 19, 2004.

References

1. E. Demaine, photos, available at `http://theory.lcs.mit.edu/~edemaine/photos/GeorgeHart_February2004`.
2. G. W. Hart, `http://www.georgehart.com`. web pages.
3. ———, *Symmetric sculpture*, J. Math. Arts **1** (2007), no. 1, 21 – 28.

COMPUTER SCIENCE DEPARTMENT, STONY BROOK UNIVERSITY, STONY BROOK, NY 11794, USA

E-mail address: `george@georgehart.com`